Topic: Definition of the derivative

Question: Use the definition of the derivative to find the derivative of the function.

$$f(x) = x^3 - 2x$$

Answer choices:

A $f'(x) = 2x^2 - 2$

B $f'(x) = 3x^2 - 2$

C $f'(x) = 3x^2 + 2$

D $f'(x) = x^2 - 1$

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Solution: B

We'll use the definition of the derivative,

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

We have to plug $x+h$ into our original function, and then plug the result back in for $f(x+h)$.

$$f(x+h)=(x+h)^3-2(x+h)$$

$$\lim_{h\to 0}\frac{\left[(x+h)^3-2(x+h)\right]-f(x)}{h}$$

Now we'll plug the original function in for $f(x)$.

$$f(x)=x^3-2x$$

$$\lim_{h\to 0}\frac{\left[(x+h)^3-2(x+h)\right]-\left(x^3-2x\right)}{h}$$

With everything plugged in, we just simplify the limit and then evaluate at $h=0$.

$$\lim_{h\to 0}\frac{(x+h)^3-2x-2h-x^3+2x}{h}$$

$$\lim_{h\to 0}\frac{\left(x^2+2xh+h^2\right)(x+h)-2x-2h-x^3+2x}{h}$$

$$\lim_{h\to 0}\frac{\left(x^3+x^2h+2x^2h+2xh^2+xh^2+h^3\right)-2h-x^3}{h}$$

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$$\lim_{h\to 0}\frac{3x^2h+3xh^2+h^3-2h}{h}$$

$$\lim_{h\to 0}3x^2+3xh+h^2-2$$

$$3x^2+3x(0)+(0)^2-2$$

$$3x^2-2$$

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Topic: Definition of the derivative

Question: Use the definition of the derivative to find the derivative of the function.

$$f(x) = x^2$$

Answer choices:

A 0

B 2

C $2x$

D $x^2 + 2x$

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Solution: C

We'll use the definition of the derivative,

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

We have to plug $x+h$ into our original function, and then plug the result back in for $f(x+h)$.

$$f(x)=(x+h)^2$$

$$\lim_{h\to 0}\frac{\left[(x+h)^2\right]-f(x)}{h}$$

Now we'll plug the original function in for $f(x)$.

$$f(x)=x^2$$

$$\lim_{h\to 0}\frac{\left[(x+h)^2\right]-x^2}{h}$$

With everything plugged in, we just simplify the limit and then evaluate at $h=0$.

$$\lim_{h\to 0}\frac{x^2+2hx+h^2-x^2}{h}$$

$$\lim_{h\to 0}\frac{2hx+h^2}{h}$$

$$\lim_{h\to 0}\frac{h(2x+h)}{h}$$

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$$\lim_{h \to 0} 2x + h$$

$$2x + 0$$

$$2x$$

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Topic: Definition of the derivative

Question: Evaluate the difference quotient.

$$f(x) = 2 - x^2 + x$$

at $a = 3$

Answer choices:

A $5 - h$

B -5

C 5

D $-h - 5$

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Solution: D

The difference quotient can be calculated using

$$\frac{f(a+h)-f(a)}{h}$$

for any function $f(x)$. If a is known, we can simplify the answer further. Using our function we get

$$\frac{f(3+h)-f(3)}{h}$$

Plugging $3+h$ and 3 into our function and substituting the results back into our difference quotient equation, we get

$$\frac{\left[2-(3+h)^2+(3+h)\right]-\left[2-(3)^2+(3)\right]}{h}$$

$$\frac{\left(2-9-6h-h^2+3+h\right)-(2-9+3)}{h}$$

$$\frac{2-9-6h-h^2+3+h-2+9-3}{h}$$

$$\frac{-h^2-5h}{h}$$

$$-h-5$$

The difference quotient of the function $f(x)=2-x^2+x$ at $a=3$ is $-h-5$.

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